42nd Finnish Summer School of Probability and Statistics
Malliavin-Skorohod calculus for additive processes with applications to Finance and Insurance

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Abstract

Malliavin-Skorohod calculus for processes with jumps is already a developed topic. The first book on this subject was the book by Bichteler, Gravereaux and Jacod [4], where Malliavin calculus was applied to the study of regularity of solutions of stochastic differential equations. Later, several other books that develop different aspects of Malliavin-Skorohod calculus for processes with jumps appeared. In particular, the book by Privault [17], where the standard Poisson case is deeply analyzed, the book by Ishikawa [11] and the book by Di Nunno, Øksendal and Proske [6], where applications to Finance are introduced for the first time.

During the last thirty years, Malliavin-Skorohod calculus has been applied to different topics in Finance and Insurance. Between them, we emphasize

a) Pricing and hedging financial derivatives

b) Computation of Greeks.

c) Analysis of the volatility surface.

d) Pricing cumulative loss derivatives.

The goal of the course is twofold. First of all, to develop a Malliavin-Skorohod type calculus for Lévy and additive processes, that extends the well-known Malliavin-Skorohod calculus for Gaussian processes. Lévy processes are processes with independent and identically distributed increments and additive processes are processes with independent increments but not necessarily stationary. In particular, we develop a non-probabilistic Malliavin-Skorohod calculus on the abstract structure of Fock space and identify the space of square integrable functionals of an additive process as a Fock space. Later we give a probabilistic interpretation of Fock space operators as operators on the canonical space of an additive process.

Secondly, we present some recent results extending the four previous topics just commented, to the framework of additive models and stochastic volatility jump diffusion models.

Applications of Malliavin-Skorohod calculus to Finance have been developed during the last thirty years. Probably the first one was due to Karatzas and Ocone in [16] where an elegant solution of the problem of

pricing and hedging financial derivatives in complete markets was found using the currently so called Clark-Hausmann-Ocone formula.

A second key application appeared in 1999 with the celebrated paper of Fournié et al. [9], where the integration by parts formula was applied successfully to improve the efficiency in computing Greeks, reducing dramatically the computational cost of this type of numerical computations.

A third interesting application was developed by Alòs in [1], where Malliavin-Skorohod calculus was applied to obtain an expansion of the pricing formula under stochastic volatility diffusion models, that allows to distinguish clearly the effect of correlation in prices. This formula, an extension of the classical Hull and White formula, allows to obtain interesting results related with the shape of the implied volatility surface, see [2], [3] and [19].

Finally, Malliavin-Skorohod calculus provides an integration by parts formula that makes pricing of derivatives based on the cumulative loss process in Insurance easier. See [10] and [12].

The goal of the second part of the course is to present results in the four previous applications, in the context of Lévy and additive processes. A survey of the first application can be found in [18] and a survey related with the third application, in [19].

To follow the course it is assumed the reader has some knowledge on Lévy processes and on Itô Stochastic Calculus in relation with Finance, like it is explained for example in [5] and [13]. Knowledge on Gaussian Malliavin calculus is recommendable. A well-known complete reference is [14]. See [15] for a short introduction.

The outline of the course is the following:

1. Introduction: Malliavin-Skorohod calculus in Finance and Insurance.
2. Lévy and additive processes. The Lévy-Itô decomposition.
3. Malliavin-Skorohod calculus without probability. The chaotic representation property.
4. A canonical space for additive processes. Malliavin-Skorohod calculus for additive processes.
5. An anticipating Itô formula.
6. Stochastic volatility jump diffusion models (SVJ).
7. Pricing and hedging financial derivatives under SVJ models
8. Sensitive Analysis for SVJ models.
9. A Hull and White formula for SVJ models.
10. Integration by parts. Pricing cumulative loss derivatives for SVJ models.

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